COMPUTER SCIENCE & ENGINEERING

QUESTION BANK

Course Title: DISCRETE MATHEMATICS
Course Code: 25CS301
Regulation: NR25

COURSE OBJECTIVES

- Introduces elementary discrete mathematics for Computer Science and Engineering.
- Topics include formal logic notation, methods of proof, induction, sets, relations, graph theory, permutations and combinations, counting principles, recurrence relations and generating functions.

COURSE OUTCOMES

- **CO1:** Ability to understand and construct precise mathematical proofs.
- **CO2:** Ability to use logic and set theory to formulate precise statements.
- **CO3:** Ability to analyze and solve counting problems on finite and discrete structures.
- **CO4:** Ability to describe and manipulate sequences.
- **CO5:** Ability to apply graph theory in solving computing problems.

UNIT – V

GRAPH THEORY

Syllabus: Basic Concepts, Isomorphism and Subgraphs, Trees and their Properties, Spanning Trees, Directed Trees, Binary Trees, Planar Graphs, Euler's Formula, Multigraphs and Euler Circuits, Hamiltonian Graphs, Chromatic Numbers, The Four-Color Problem.

PART – A

SHORT ANSWER QUESTIONS

S.No.	Questions	BT	CO	PO
1	Define a graph. Explain the terms vertex, edge, order and size of a graph.	L1	CO5	PO1
2	Define a directed graph and a directed edge.	L1	CO5	PO1
3	Define a simple graph, multigraph and pseudograph.	L1	CO5	PO1
4	Define the degree of a vertex. State the Handshaking Theorem.	L1	CO5	PO1
5	Define a complete graph and a complete bipartite graph with suitable examples.	L1	CO5	PO1
6	Define a subgraph and a spanning subgraph.	L1	CO5	PO1
7	Define isomorphic graphs. State any two necessary conditions for two graphs to be isomorphic.	L1	CO5	PO1
8	Define a tree and state any two properties of a tree.	L1	CO5	PO1
9	Define a spanning tree and a minimum spanning tree.	L1	CO5	PO1
10	Define an Euler path and an Euler circuit.	L1	CO5	PO1
11	Define a Hamiltonian path and a Hamiltonian cycle.	L1	CO5	PO1
12	Define a planar graph and give one example each of a planar and a non-planar graph.	L1	CO5	PO1
13	State Euler's formula for a connected planar graph.	L1	CO5	PO1
14	Define the chromatic number of a graph.	L1	CO5	PO1
15	State the Four-Color Theorem.	L1	CO5	PO1
16	Define the indegree and outdegree of a vertex in a directed graph.	L1	CO5	PO1

PART – B

LONG ANSWER QUESTIONS

S.No.	Questions	BT	CO	PO
17	Let D be the digraph with vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$ and directed edge set $E = \{(v_1, v_4), (v_2, v_3), (v_3, v_5), (v_4, v_2), (v_4, v_4), (v_4, v_5), (v_5, v_1)\}$. Draw the digraph and determine the indegree and outdegree of each vertex.	L3	CO5	PO2
18	A graph contains 16 edges and every vertex has degree 4. Determine the number of vertices in the graph.	L3	CO5	PO2
19	A graph has 21 edges, three vertices of degree 4 and all the remaining vertices of degree 3. Determine the number of vertices in the graph.	L4	CO5	PO2
20	Let G be a simple graph of order n having 56 edges. If the complement of G has 18 edges, determine n.	L3	CO5	PO2
21	Prove that the complement of a bipartite graph need not be bipartite. Give a suitable example.	L4	CO5	PO2
22	Draw the complete bipartite graph $K_{3,4}$ and prove that it does not contain a Hamiltonian cycle.	L4	CO5	PO2
23	Show that the complete bipartite graph $K_{3,3}$, also known as Kuratowski's second graph, is non-planar.	L4	CO5	PO2
24	Prove that a connected graph G remains connected after removing an edge e if and only if e belongs to some cycle of G.	L4	CO5	PO2
25	Using the Depth First Search (DFS) method, determine a spanning tree for a given connected graph, taking a specified vertex as the root.	L3	CO5	PO2
26	Using the Breadth First Search (BFS) method, determine a spanning tree for a given connected graph, taking a specified vertex as the root.	L3	CO5	PO2
27	Prove that every tree with n vertices has exactly $n - 1$ edges.	L4	CO5	PO2
28	Prove that a connected graph with n vertices and $n - 1$ edges is a tree.	L4	CO5	PO2
29	Let G be a connected planar graph with v vertices, e edges and f faces. Prove Euler's formula $v - e + f = 2$.	L4	CO5	PO2
30	Determine the number of faces of a connected planar graph having 10 vertices and 15 edges using Euler's formula.	L3	CO5	PO2
31	Determine whether the following graphs have an Euler path or Euler circuit. Justify your answer using the degrees of the vertices.	L4	CO5	PO2

S.No.	Questions	BT	CO	PO
32	Determine whether a given graph is Hamiltonian. If it is Hamiltonian, identify a Hamiltonian cycle.	L4	CO5	PO2
33	Prove that every complete graph K_n, for $n \geq 3$, is Hamiltonian.	L4	CO5	PO2
34	Determine the number of edges in the following graphs: (i) K_n, (ii) $K_{m,n}$, and (iii) P_n.	L3	CO5	PO2
35	Show that two given graphs are not isomorphic by identifying suitable graph invariants.	L4	CO5	PO2
36	Determine whether the two given graphs are isomorphic. If they are isomorphic, construct an isomorphism between their vertex sets.	L4	CO5	PO2
37	Find the chromatic number of the following graphs: (i) K_n, (ii) C_n, and (iii) $K_{m,n}$.	L3	CO5	PO2
38	Find the chromatic number of a given graph using a suitable vertex-coloring procedure.	L3	CO5	PO2
39	Explain the Four-Color Problem and state the Four-Color Theorem. Discuss its significance in graph coloring and map coloring.	L2	CO5	PO1
40	Show that every planar graph can be colored using at most four colors. State the Four-Color Theorem and explain its application to map coloring.	L4	CO5	PO2
41	Explain directed trees and binary trees with suitable examples. Distinguish between a rooted tree and a binary tree.	L2	CO5	PO1
42	For a given rooted binary tree, determine its preorder, inorder and postorder traversals.	L3	CO5	PO2
43	Explain the difference between Eulerian and Hamiltonian graphs. Give suitable examples of each.	L2	CO5	PO1
44	Let G be a graph of order 9 in which every vertex has degree 5 or 6. Prove that at least 5 vertices have degree 6 or at least 6 vertices have degree 5.	L4	CO5	PO2

BLOOM'S TAXONOMY LEVELS (BT)

Level	Bloom's Taxonomy	Description
L1	Remembering	Define, State, List, Identify, Recall
L2	Understanding	Explain, Describe, Interpret, Classify, Summarize
L3	Applying	Solve, Calculate, Construct, Demonstrate, Use

Level	Bloom's Taxonomy	Description
L4	Analyzing	Analyze, Differentiate, Examine, Compare, Prove
L5	Evaluating	Justify, Critique, Assess, Validate, Defend
L6	Creating	Design, Formulate, Develop, Generate, Construct

CO–PO ALIGNMENT FOR UNIT-V

CO5

Ability to apply graph theory in solving computing problems.

Unit-V questions assess the ability to:

- Understand basic graph-theoretic terminology.
- Represent graphs using vertices and edges.
- Analyze directed and undirected graphs.
- Determine degrees of vertices.
- Identify subgraphs and isomorphic graphs.
- Analyze trees and their properties.
- Construct spanning trees.
- Apply DFS and BFS techniques.
- Analyze planar and non-planar graphs.
- Apply Euler's formula.
- Determine Euler paths and Euler circuits.
- Analyze Hamiltonian paths and cycles.
- Determine chromatic numbers.
- Apply graph coloring concepts to computing and map-coloring problems.

PO MAPPING

PO1 – Engineering Knowledge:
Apply mathematical and graph-theoretic concepts to solve problems in Computer Science and Engineering.

PO2 – Problem Analysis:
Analyze graph structures, identify relevant properties and apply suitable graph-theoretic techniques to solve problems.

UNIT-V TOPIC-WISE COVERAGE

Unit-V Topic	Question Numbers
Basic Concepts of Graphs	1, 3, 4, 5
Directed Graphs	2, 16, 17
Degree of Vertices	4, 18, 19, 44
Complete Graphs	5, 34, 37
Bipartite Graphs	5, 21, 22, 23, 34, 37
Subgraphs	6
Isomorphic Graphs	7, 35, 36
Trees	8, 27, 28
Spanning Trees	9, 25, 26
Directed Trees	41
Binary Trees	41, 42
Planar Graphs	12, 23, 29, 30, 40
Euler's Formula	13, 29, 30
Euler Paths and Circuits	10, 31, 43
Multigraphs	3
Hamiltonian Graphs	11, 22, 32, 33, 43
Chromatic Numbers	14, 37, 38
Graph Coloring	38, 39, 40
Four-Color Problem	15, 39, 40

IMPORTANT CORRECTIONS MADE FROM THE ORIGINAL QUESTION BANK

1. **“Define directed graph and directed edge set”** is correctly classified as **L1 – Remembering**.

2. **“Draw a diagram of the graph…”** was changed from L1 to **L3 – Applying**, because the student has to construct a graph from given data.

3. **“What is meant by vertices and edges”** was changed from L4 to **L1**, because it asks for basic definitions.

4. **“Show that…” questions** were changed from L1/L2 to **L4 – Analyzing**, because proving a graph-theoretic property requires analysis and reasoning.

5. **Degree-counting problems** were changed to **L3/L4**, because students must apply the Handshaking Theorem and analyze degree information.

6. **Hasse diagrams are not part of Unit-V**; therefore lattice/Hasse-diagram questions have not been included here.

7. **Euler's Formula** was added because it is explicitly mentioned in the NR25 Unit-V syllabus.

8. **Chromatic Numbers** were added because they are explicitly mentioned in the syllabus.

9. **The Four-Color Problem** was added because it is explicitly mentioned in the syllabus.

10. **Directed Trees and Binary Trees** were added because they are explicitly mentioned in the syllabus.

11. **DFS/BFS spanning-tree questions** have been included under spanning trees and graph traversal applications.

12. **Kuratowski's second graph** is correctly identified as $K_{3,3}$.

13. The question concerning “the size of G is 56 and the size of G is 18” was corrected to mean **the graph has 56 edges and its complement has 18 edges**, which is mathematically meaningful.

14. The question **“Define a spanning tree and a minimal spanning tree”** was retained as a basic concept but the term **minimum spanning tree** is used consistently.

15. Questions have been mapped primarily to **CO5**, since CO5 explicitly states the application of graph theory.

RECOMMENDED BT DISTRIBUTION FOR UNIT-V

BT Level	Focus
L1 – Remembering	Definitions, terminology and basic graph concepts
L2 – Understanding	Explanation and interpretation of graph concepts

BT Level	Focus
L3 – Applying	Graph construction, degree calculations, DFS/BFS, coloring and Euler formula applications
L4 – Analyzing	Proofs, isomorphism, planarity, Hamiltonian properties and graph-theoretic analysis

TEXT BOOKS

1. **Discrete Mathematical Structures with Applications to Computer Science** – J. P. Tremblay and R. Manohar, McGraw-Hill, 1st Edition.
2. **Discrete Mathematics for Computer Scientists and Mathematicians** – Joe L. Mott, Abraham Kandel and Theodore P. Baker, Prentice Hall of India, 2nd Edition.

REFERENCE BOOKS

1. **Discrete and Combinatorial Mathematics – An Applied Introduction** – Ralph P. Grimaldi, Pearson Education, 5th Edition.
2. **Discrete Mathematical Structures** – Thomas Koshy, Tata McGraw-Hill.